Introduction to AI

Lecture 13

Inference Rules

Dr. Tamal Ghosh
Department of CSE
Adamas University

Definition of Inference

In artificial intelligence, we need intelligent computers that can create new logic from old logic or by evidence, **so generating conclusions from evidence and facts is termed as Inference**.

Inference rules:

Inference rules are the templates for generating valid arguments. Inference rules are applied to derive proofs in artificial intelligence, and the proof is a sequence of conclusions that leads to the desired goal.

In inference rules, the implication among all the connectives plays an important role. Following are some terminologies related to inference rules:

Rules of Inference

- **Implication:** It is one of the logical connectives which can be represented as P → Q. It is a Boolean expression.
- **Converse:** The converse of implication, which means the right-hand side proposition goes to the left-hand side and vice-versa. It can be written as Q → P.
- **Contrapositive:** The negation of converse is termed as contrapositive, and it can be represented as ¬ Q → ¬ P.
- **Inverse:** The negation of implication is called inverse. It can be represented as ¬ P → ¬ Q.

Truth Table from Inference Rule

- From the above term some of the compound statements are equivalent to each other, which we can prove using truth table:

P	Q	P → Q	Q→ P	¬ Q → ¬ P	¬ P → ¬ Q.
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

- Hence from the above truth table, we can prove that P → Q is equivalent to ¬ Q → ¬ P, and Q→ P is equivalent to ¬ P → ¬ Q.

Types of Inference rules

1. Modus Ponens:

The Modus Ponens rule is one of the most important rules of inference, and it states that if P and P → Q are true, then we can infer that Q will be true. It can be represented as:

Notation for Modus ponens: $$\frac{P \rightarrow Q, \quad P}{\therefore Q}$$

Example:

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q
Statement-2: "I am sleepy" ==> P
Conclusion: "I go to bed." ==> Q.
Hence, we can say that, if P→ Q is true and P is true then Q will be true.

P	Q	$P \rightarrow Q$
0	0	0
0	1	1
1	0	0
1	1	1

Types of Inference rules

2. Modus Tollens:

The Modus Tollens rule states that if P→ Q is true and **¬ Q is true, then ¬ P** will also be true. It can be represented as:

Notation for Modus Tollens: $$\frac{P \rightarrow Q, \quad \sim Q}{\sim P}$$

Statement-1: "If I am sleepy then I go to bed" ==> P→ Q
Statement-2: "I do not go to bed."==> ~Q
Statement-3: Which infers that "**I am not sleepy**" => ~P
Proof by Truth table:

P	Q	$\sim P$	$\sim Q$	$P \rightarrow Q$
0	0	1	1	1 ←
0	1	1	0	1
1	0	0	1	0
1	1	0	0	1

Types of Inference rules

3. Hypothetical Syllogism:

The Hypothetical Syllogism rule state that if P→R is true whenever P→Q is true, and Q→R is true. It can be represented as the following notation:

Example:

Statement-1: If you have my home key then you can unlock my home. **P→Q**
Statement-2: If you can unlock my home then you can take my money. **Q→R**
Conclusion: If you have my home key then you can take my money. **P→R**

P	Q	R	$P \to Q$	$Q \to R$	$P \to R$
0	0	0	1	1	1 ←
0	0	1	1	1	1 ←
0	1	0	1	0	1
0	1	1	1	1	1 ←
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	1	1 ←

Types of Inference rules

4. Disjunctive Syllogism:

The Disjunctive syllogism rule state that if PVQ is true, and ¬P is true, then Q will be true. It can be represented as:

Notation of Disjunctive syllogism: $\frac{P \vee Q,\ \neg P}{Q}$

Example:

Statement-1: Today is Sunday or Monday. ==>PVQ

Statement-2: Today is not Sunday. ==> ¬P

Conclusion: Today is Monday. ==> Q

P	Q	$\neg P$	$P \vee Q$
0	0	1	0
0	1	1	1 ←
1	0	0	1
1	1	0	1

Types of Inference rules

5. Addition:

The Addition rule is one of the common inference rule, and it states that If P is true, then PVQ will be true.

Notation of Addition: $\frac{P}{P \vee Q}$

Example:

Statement: I have a vanilla ice-cream. ==> P

Statement-2: I have Chocolate ice-cream.

Conclusion: I have vanilla or chocolate ice-cream. ==> (PVQ)

P	Q	$P \vee Q$
0	0	0
1	0	1 ←
0	1	1
1	1	1 ←

Types of Inference rules

6. Simplification:

The simplification rule state that if **P∧ Q** is true, then **Q or P** will also be true. It can be represented as:

Notation of Simplification rule: $\frac{P \land Q}{Q}$ Or $\frac{P \land Q}{P}$

P	Q	$P \land Q$
0	0	0
1	0	0
0	1	0
1	1	1

Types of Inference Rules

7. Resolution:

The Resolution rule states that if P∨Q and ¬ P∧R is true, then Q∨R will also be true. **It can be represented as**

Notation of Resolution = $\frac{A \lor B, \neg A \lor C}{B \lor C}$

A	B	¬B	C	A∨B	¬B∨C	A∨C
1	0	1	1	1	1	1
1	1	0	1	1	1	1
0	1	0	1	1	1	1
0	0	1	1	0	1	1
1	0	1	0	1	1	1
1	1	0	0	1	0	1
0	1	0	0	1	0	0
0	0	1	0	0	1	0